AFIT/GA/AA/78D-2

# MATERIAL DAMPING AS A MEANS OF QUANTIFYING FATIGUE DAMAGE IN COMPOSITES

Thesis

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2nd Lt USAF

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MATERIAL DAMPING

AS A MEANS OF

QUANTIFYING FATIGUE DAMAGE

IN COMPOSITES,

Thesis (9) 11. 7. the 1-

Presented to the Faculty of the School of Engineering of the Air Force Institute of Technology

Air University (ATC)

In Partial Fulfillment of the

Requirement for the Degree of

Master of Science

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#### **Preface**

It is suggested that debonding and fiber fracture in composite materials might lead to significant internal energy losses. In the past, this damage has been assessed primarily by nondestructive techniques such as ultrasonics and radiography. This was an experimental study to test the feasibility of using the measurement of internal energy loss as a nondestructive technique to determine the amount of damage that has occurred in a composite during a fatigue test. I performed fatigue tests on graphite—epoxy composite specimens in order to quantify the damage in terms of the damping characteristics of the composite.

I am indebted to Dr. George Sendeckyj of the Air Force Flight

Dynamics Laboratory for his support and assistance in providing the

specimens required for this investigation; to Ed Porter and Ron Selner

of the Air Force Materials Laboratory for performing damage inspection

through radiography and ultrasonics.

I am most appreciative of Capt. Daniel Hayes, AFIT Mechanics

Laboratory Engineer, for his invaluable technical assistance and

support. I am deeply grateful to Dr. Peter Torvik, my thesis advisor,

who provided guidance and encouragement throughout this study.

To Gary Andreoletti, I offer my thanks for his patience and understanding throughout this study.

Cynthia A. Bourne

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### Symbols

<b>A</b>	cross-sectional area of beam, in <sup>2</sup>
$\mathbf{a_T}$	Maximum tip acceleration, in/sec <sup>2</sup>
c	specimen thickness, in
<b>b</b> T	maximum tip amplitude, in
E	Young's Modulus, 1b/in <sup>2</sup>
E <sub>1</sub> , E <sub>2</sub>	modulus of composite in longitudinal and transverse directions of fiber axes, respectively, 1b/in <sup>2</sup>
EI	stiffness, lb-in
G <sub>12</sub>	shear modulus in x-y plane, lb/in <sup>2</sup>
g	gravitational acceleration, ft/sec <sup>2</sup>
1	effective beam length from root to tip, in
ı	moment of inertia, in4
M	bending moment, 1b-in
N	number of cycles
<b>p</b> .	frequency of vibration, rad/sec
Q	shear force, 1b
x, y, z	coordinate axes and displacements, in, along those axes
η	coefficient of damping
$ u_{ m R}$	resonant frequency, cps
$v_{12}, v_{21}$	Poisson's ratio
ρ	material density, slugs/in <sup>3</sup>
σ	stress, 1b/in <sup>2</sup>
$\sigma_{\mathrm{R}}$ , $\sigma_{\mathrm{max}}$	stresses at root and edge of hole, respectively, 1b/in <sup>2</sup>

#### Abstract

The purpose of this experimental study was to investigate the possibility of quantifying damage in composites in terms of damping characteristics. Ben'ing fatigue tests were performed on  $((0/\frac{1}{2}45/90)_{\rm S})_2 \ {\rm T300/5208} \ {\rm graphite-epoxy} \ {\rm composite} \ {\rm specimens.} \ {\rm Testing} \ {\rm was} \ {\rm performed} \ {\rm by} \ {\rm exciting} \ {\rm a} \ {\rm cantilever} \ {\rm beam} \ {\rm so} \ {\rm as} \ {\rm to} \ {\rm maintain} \ {\rm a} \ {\rm constant} \ {\rm amplitude} \ {\rm at} \ {\rm resonant} \ {\rm frequency}. \ {\rm Results} \ {\rm indicate} \ {\rm that} \ {\rm the} \ {\rm amount} \ {\rm of} \ {\rm damage} \ {\rm occurring} \ {\rm in} \ {\rm this} \ {\rm composite} \ {\rm can} \ {\rm be} \ {\rm explained} \ {\rm in} \ {\rm terms} \ {\rm of} \ {\rm an} \ {\rm increase} \ {\rm in} \ {\rm its} \ {\rm material} \ {\rm damping}. \ {\rm The} \ {\rm resonant} \ {\rm frequency} \ {\rm was} \ {\rm found} \ {\rm to} \ {\rm decrease} \ {\rm as} \ {\rm the} \ {\rm material} \ {\rm damage} \ {\rm increased}.$ 

# MATERIAL DAMPING AS A MEANS OF QUANTIFYING FATIGUE DAMAGE IN COMPOSITES

#### I. Introduction

#### Background

Composite materials are ideal for structural applications where high strength-to-weight and stiffness-to-weight ratios are required. Aircraft and spacecraft are typical weight-sensitive structures in which composite materials are cost-effective.

Composites designated as advanced fiber-reinforced composite materials are those with new ultrahigh strength and stiffness fibers such as boron and graphite. The matrix material can be a plastic such as epoxy or a metal such as aluminum. Advanced composites have two major advantages: improved strength and stiffness.

Composites can be made three times as strong as aluminum yet weigh only 60 percent as much. Other composites can be made that have the same strength and stiffness as high-strength steel, but are 70 percent lighter. Moreover, composite materials can be tailored to efficiently meet design requirements of strength, stiffness, and other parameters. These advantages should lead to new and more efficient aircraft and spacecraft designs in contrast to those based on conventional materials.

The strength of composites is governed by their flaw-initiated characteristics. Thus, the mechanics of fracture including crack propagation or extension are of extreme importance in the design analysis of composite structures. The fracture process takes place in three stages. First, an initial flaw may be present or a microcrack initiated. Second, the microcrack grows and may link with

others to attain microcrack size. Third, at a critical stress level, the crack propagates through the material. Fracture occurs when the crack reaches a critical length.

Salkind (Ref 13) has proposed that composites may fail by matrix cracking, fiber failure, and delamination, either separately or in combination. He also suggests that a fatigue failure criterion could be the number of cycles to a given change in stiffness rather than the number of cycles to fracture since the loss of stiffness can result in structural failure long before complete fracture occurs.

The internal damping of fiber-reinforced composites is more sensitive than stiffness to the failure mechanisms previously mentioned. Accordingly, damping measurements can be valuable in the early detection of structural damage.

Much of the experimental work on the damping and stiffness properties of composite materials has been concerned with measuring frequency and termperature dependence (Ref 4). Both aluminum and fiber-reinforced plastic beams have been tested by the methods of logarithmic decrement and resonant dwell (Ref 8). Air damping was shown to be significant at low frequencies and large amplitudes. Schultz and Tsai (Ref 14) used logarithmic decrement and bandwidth techniques to test aluminum and glass fiber-reinforced plastics in flexural vibration. They found that the amount of fatigue cracking correlated reasonably well with the amount of increase in damping. Ulitchny and Lucas (Ref 10) studied the change in the damping of free vibrations and found crack damage present as changes in damping occurred.

For the commercial utilization of composites, it is necessary to ensure material integrity through reliable methods. Damage

evaluation by nondestructive techniques (NDT) is currently being investigated. Of the NDT methods available, holography and ultrasonics show the most promise. Because damping can be expected to be sensitive to microstructural damage, damping measurements may provide a means of achieving the necessary quality control.

This thesis studied the feasibility of using the change in material damping to quantify structural damage in the course of a destructive bending fatigue test on composite beams. Observed changes in damping were compared with NDT results in order to provide a comparison between their two means of evaluating damage.

#### II. Analysis of Beam Stress and Displacement

#### Beam Theory

The composite specimens were modeled as beams and, as such, a few simplifying assumptions have been made:

- 1) Beam deformations are described by the elementary theory of flexure, so that plane sections normal to beam axis remain plane and normal to the deformed axis
- 2) Each ply is assumed to satisfy the linear, elastic constitutive relationship
- 3) Plies are assumed to have uniform thickness

The well-known differential equation of the differential equation of the differential curve of a beam whose vibrations occur in its plane of symmetry (1.5) is

$$EI \frac{d^2y}{dx^2} = -M \tag{1}$$

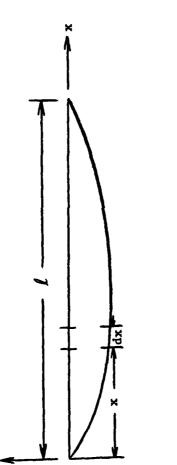
in which

y(x) = deflection of the neutral axis,

EI = stiffness, and

M = bending moment at any cross-section. The direction of the axis and positive directions of bending moments and shear forces are shown in Figure 1.

Differentiating Eq (1) twice gives



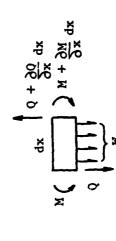


Figure 1. Bending Moment, Shear Force, Axis Directions

$$\frac{d}{dx} \quad \text{EI}\left(\frac{d^2y}{dx^2}\right) = \frac{dM}{dx} = Q \quad \text{and}$$

$$\frac{d^2}{dx^2} \quad \text{EI}\left(\frac{d^2y}{dx^2}\right) = \frac{dQ}{dx} = -w \quad (2)$$

The continuous load of intensity, w, can be expressed as

$$\rho_{\rm A} = \frac{\partial^2 y}{\partial t^2} \tag{3}$$

Where ho is the mass of beam material per unit volume and A is the cross-sectional area.

Letting  $w = \rho A \frac{\partial^2 y}{\partial t^2}$  is Eq (2), the general equation for

lateral vibration of a beam becomes

$$\frac{\partial^2}{\partial x^2} = \frac{\partial^2 y}{\partial x^2} = -\rho A \frac{\partial^2 y}{\partial t^2}$$

When EI remains constant along the length of the beam, this equation becomes

$$EI \frac{\partial^4 y}{\partial x^4} = - \rho A \frac{\partial^2 y}{\partial t^2}$$

or

$$\frac{\partial^2 y}{\partial t^2} + a^2 \frac{\partial^4 y}{\partial x^4} = 0 \tag{4}$$

where 
$$a^2 = \frac{EI}{A\rho}$$
 . (4a)

The deflection of a beam at any location varies harmonically

with time and can be expressed as

$$y = X(A cospt + B sinpt)$$
 (5)

Where X is a function of the coordinate x. Substituting Eq (5) into Eq (4) leads to

$$\frac{d^4X}{dx^4} = \frac{p^2}{a^2} X \tag{6}$$

Using the notation 
$$\frac{p^2}{a^2} = p^2 \frac{\rho A}{EI} = k^4$$
 (7)

it can be verified that sin kx, cos kx, sinh kx, and cosh kx are all solutions of Eq (6) and that the general solution can be written in the form

$$X = C_1(\cos kx + \cosh kx) + C_2(\cos kx - \cosh kx) + C_3(\sin kx + \sinh kx) + C_4(\sin kx - \sinh kx)$$
(8)

where  $^{C}1$ ,  $^{C}2$ ,  $^{C}3$ , and  $^{C}4$  are constants to be determined from the end conditions of the beam.

Assuming that one end (x = 0) is built in and the other end is free, the following end conditions are obtained:

$$(x)_{x=0} = 0 (a)$$

$$\left(\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\mathbf{x}}\right)_{\mathbf{x}=\mathbf{0}} = \mathbf{0} \tag{b}$$

$$\left(\frac{d^2X}{dx^2}\right)_{x=1} = 0 \tag{c}$$

$$\left(\frac{d^3x}{dx^3}\right)_{x=1} = 0 \tag{d}$$

The first two conditions result in  $C_{1} = C_{3} = 0$ . The other two give the following equation from which the frequency can be obtained:

(cosk1)(coshk1) = -1

When a tip mass is added to the previously free end of the beam, new end conditions are obtained:

$$(X)_{x=0} = 0 (e)$$

$$\left(\frac{\mathrm{dX}}{\mathrm{dx}}\right)_{\mathrm{x}=0} = 0 \tag{f}$$

$$\left(\frac{\mathrm{d}^2 X}{\mathrm{d} x^2}\right)_{x=1} = 0 \tag{g}$$

$$\left(\frac{d^3X}{dx^3}\right)_{x=1} = \frac{m}{EI} X(1) \left(-p^2\right)$$
 (h)

where m = tip mass. The rotatory inertia of the tip mass has been neglected. Again, the first two conditions give  $^{\rm C}_{1}$ =  $^{\rm C}_{3}$ = 0 in Eq (8). The last two give the following equation:

$$\frac{(\cosh 1)(\cosh k1) + 1}{(\sinh k1)(\cosh k1) - (\cosh k1)(\sinh k1)} = \frac{p^2_m}{k^3 EI}$$
(9)

From Eqs (7) and (4a)

$$p^2 = \frac{EIk^4}{\rho A} \tag{9a}$$

and Eq (9) can be expressed as

$$\frac{(\cosh 1)(\cosh k1) + 1}{(\sinh k1)(\cosh k1) - (\cosh k1)(\sinh k1)} = k1 \left(\frac{m}{\rho A1}\right)$$
(9b)

This transcendental equation, together with Eq (9a), must be solved in order to find the natural frequencies in terms of the mass ratio of tip mass to beam.

An approximate method known as Rayleigh's Quotient can also be used to estimate the frequency of vibration. Its application comes from consideration of the total energy of the system. In applying this method, an initial assumption has to be made regarding the beam deflection shape during vibration. In this case, the deflection was taken to be

$$X = f(x) = x^{2}(31-x)$$
 (10)

The potential energy of the beam is simply its strain energy:

$$\int_0^1 \frac{EI}{2} \left( \frac{d^2 y}{dx^2} \right)^2 dx = \int_0^1 \frac{EI}{2} \left( \frac{d^2 X}{dx^2} \right)^2 (sinpt)^2 dx$$

As the tip mass is assumed to contribute no potential energy, the maximum potential energy is then

$$\int_0^1 \frac{EI}{2} \left( \frac{d^2 x}{dx^2} \right)^2 dx \tag{11}$$

The kinetic energy in the beam is

$$\int_0^1 \frac{\rho A}{2} \left(\frac{dy}{dt}\right)^2 dx = \int_0^1 \frac{\rho A \rho}{2} x^2 \cos^2 \rho t dx$$

This gives a maximum kinetic energy of

$$\int_{a}^{1} \frac{\rho A}{2} p^{2} \chi^{2} dx \tag{12}$$

for the beam. The tip mass contributes to the kinetic energy of the system:

$$\frac{m}{2} \left( \frac{dy}{dt} \right)_{x=1}^{2} = \frac{m}{2} \left( p^{2} x^{2} \right)_{x=1} \cos^{2} pt$$

Then the maximum kinetic energy of the tip mass is

$$\frac{m}{2} p^2 \chi^2 \Big|_{\chi=1} \tag{13}$$

The total energy of the system is a constant and equal to the sums of the kinetic and potential energies. It follows that the maximum kinetic and potential energies of the system can be equated. This yields:

$$\int_{0}^{1} \frac{EI}{2} \left( \frac{d^{2}X}{dx^{2}} \right)^{2} dx = \int_{0}^{1} \frac{\rho A}{2} p^{2}X^{2} dx + \frac{m}{2} p^{2}X^{2} \Big|_{x=1}$$

Rearranging, the frequency can be found to be:

$$p^{2} = \frac{\int_{0}^{1} \frac{EI}{2} \left(\frac{d^{2}X}{dx^{2}}\right)^{2} dx}{\int_{0}^{1} \frac{\rho AX^{2}}{2} dx + \frac{m}{2} X(1)^{2}}$$
(14)

Substituting Eq (10) into Eq (14) gives the frequency in terms of

the assumed beam deflection:

$$p^{2} = \frac{\int_{0}^{1} \frac{EI}{2} (61-6x)^{2} dx}{\int_{0}^{1} \frac{\rho A}{2} x^{4} (31-x)^{2} dx + \left[\frac{m}{2} x^{2} (31-x)^{2}\right]_{x=1}}$$

carrying out the integration yields

$$p^2 = \frac{210 \text{ ET}}{33 \rho \text{A1}^4 + 70 \text{m1}^3} \tag{15}$$

In both Rayleigh's Quotient and the exact solution,  $p^2$  can be solved for in terms of the tip and beam masses. The necessary material constants for the material used in this investigation were determined and are listed in Table I. Eq (9) can be solved for kl by an iterative process and the resonant frequency,  $\nu_R$ , can be determined from Eq (9a). This yields p = 174.96 rad/sec or  $\nu_R = 27.85$  cps for exact solution. Using Rayleigh's method, p=171.62 rad/sec or  $\nu_R = 27.31$  cps. There is less than two percent error in the frequency determined by both methods.

#### Root Stress and Tip Amplitude

Since the experimental procedure employed in this investigation used the measurement of tip displacement as a means of controlling beam stress, it was necessary to develop a relationship between stresses and tip displacements. Since the beam was assumed to be in simple bending, nominal beam stresses were taken to be

$$\sigma = \frac{Mc}{T}$$

Incorporating Eqs (1) and (5), the maximum outer fiber stress at any point along the beam is

Table I

Specimen Material Constants					
Young's Modulus, E <sup>1</sup>	1.138 x 10 <sup>7</sup> psi				
Area Moment of Inertia, I	5.1177 x 10 <sup>-5</sup> in <sup>4</sup>				
Stiffness, EI	$5.8233 \times 10^2$ lb-in				
Beam thickness, 2c	0.085 in				
Beam width, D	1.00 in				
Effective beam length from root to tip mass, 1	5.12 in				
Beam Mass, PA1	7.734 x 10 <sup>-4</sup> slugs				
Tip Mass, m <sub>T</sub> includes accelero- meter, fiberglass doublers, beam tip beyond center of accelerometer	4.446 x 10 <sup>-3</sup> slugs				
Mass Ratio, $\frac{m}{\rho A1}$	5.749				

1. Young's Modulus was computed by the method described in Appendix A

$$\sigma = cE \frac{d^2X}{dx^2}$$
 (Acospt + Bsinpt)

The stress at the root (x=0) is then

$$\sigma_{\rm R} = cE \frac{d^2 X(0)}{dx^2}$$
 (Acospt + Bsinpt) (16)

Acceleration of the tip can be expressed as

$$a_{T} = \frac{d^{2}y}{dt^{2}} = p^{2}X(Acospt + Bsinpt)$$

$$x=1$$
(17)

Since the ratio of root curvature to tip displacement is determined solely by the mode shape, it is convenient to combine Eqs (16) and (17) into

$$\sigma_{R} = \frac{cE}{p^2} a_{T} \frac{X''(0)}{X(1)}$$
 (18)

Using the approximate mode shape, Eq (10), we find

$$\sigma_{R} = \frac{a_{T}}{p^{2}} \quad cE \frac{f''(0)}{f(1)}$$
 (19)

Since the tip amplitude is related to the acceleration through

$$\boldsymbol{\delta}_{\mathrm{T}} = \frac{\mathbf{a}_{\mathrm{T}}}{2} \tag{20}$$

the tip amplitude required to produce a desired stress at the root is

$$\delta_{T} = \frac{\sigma_{R} f(1)}{cEf''(0)} = \frac{\sigma_{R}}{cE} \frac{1^{2}}{3}$$
 (21)

Using the exact mode shape, Eq (8), with  $C_1 = C_3 = 0$  and

$$\frac{c_2}{c_2} = \frac{-c_4(\sinh + \sinh kl)}{\cosh l + \cosh kl}$$
 (22)

results in

$$X(x) = C_4 \left[ \frac{(\sinh x - \sinh k1) - (\sinh k1 + \sinh k1)(\cosh x - \cosh kx)}{\cosh k1} \right]$$
 (23)

Thus

$$X(1) = 2C_4 \frac{\left(\sinh(1)(\cosh(1) - (\cosh(1)(\sinh(1)))\right)}{\cosh(1 + \cosh(1))}$$
(24)

and

$$X^{n}(0) = 2C_{4}k^{2} \left[ \frac{\sinh 1 + \sinh k1}{\cosh 1 + \cosh k1} \right]$$

Thus

$$\sigma_{R} = \frac{cE(T)}{p^{2}}k^{2} \left[ \frac{\sinh 1 + \sinh k1}{(\sinh 1)(\cosh k1) - (\cosh 1)(\sinh k1)} \right]$$

For the beams used in this investigation, Eq (9b) was found to yield a value of kl = .84142 or

$$\delta_{T} = \frac{\sigma_{R}}{cE} \left[ \frac{(\sinh 1)(\cosh 1) - (\cosh 1)(\sinh 1)}{k^{2} (\sinh 1 + \sinh 1)} \right]$$

OT

$$\delta_{\rm T} = .3312 \frac{\sigma_{\rm R}^2}{cE}$$

The formula based on the approximate mode shape differs from this by less than one percent.

For a beam having the properties given in Table I,

$$\sigma_{R} = 55,713 \delta_{T} \text{ psi/in}$$

Inclusion of a hole centered a distance b from the root leads to stresses at the hole edge (Ref 12) given by

$$\sigma_{\text{max}} = f \frac{6M(b)}{c^2(D-2r)}$$
 (25)

Where 
$$f = \begin{bmatrix} 1.79 + \frac{.25}{1 + \frac{2r}{c}} + \frac{.81}{1 + (\frac{2r}{c})^2} - \frac{.26}{1 + (\frac{2r}{c})^3} \end{bmatrix}$$

$$x \left[ 1 - 1.04 \frac{2r}{D} + 1.22 \left( \frac{2r}{D} \right)^2 \right] \tag{26}$$

r = hole radius

D = width of beam

2c = thickness of beam

M = moment at hole = EIy"

The stress at the hole can be expressed in terms of the nominal stress where the nominal stress is

$$\sigma_n = \frac{M(b)c}{I} = \frac{6M(b)}{Dc^2}$$

The stress concentration factor is found to be

$$\frac{\sigma_{\text{max}}}{\sigma_{\text{n}}} = \frac{f}{1 - \frac{2r}{D}}$$
 (27)

and has a value of 2.0701 for r=0.098 in, D=1.00 in, and c=0.085 in.

The nominal stress can be written in terms of the tip amplitude since

$$M = EIX''(hole)$$

so that

$$M(b) = EI \frac{d^2X(b)}{dx^2}$$

Substituting the exact mode shape given by Eq (23) and eliminating  ${\rm C}_{L}$  through the use of Eq (24), the nominal stress is found to be

$$\sigma_{\rm n} = \text{Ec } \delta_{\rm T} k^2 \left[ \frac{(\text{sinkb-sinhkb})(\text{coskl+coshkl}) - (\text{sinkl+sinhkl})}{2[(\text{sinkl})(\text{coshkl}) - (\text{coskl})(\text{sinhkl})]} \right]$$

For a beam having the properties in Table I,

$$\sigma_{\rm n}$$
 = 25,089  $\delta_{\rm T}$  psi/in

Combining this with the stress concentration factor given by Eq (27), the stress at the edge of the hole is found to be

$$\sigma_{\text{max}} = 52,384 \delta_{\text{m}} \text{ psi/in}$$

where  $\boldsymbol{\delta}_{\mathrm{T}}$  is the tip displacement. This value is seen to be slightly less than the stress at the root of the beam; however, failure may be expected to initiate at the hole edge because of inter-laminar edge stresses (Ref 14) and imperfections generated by the process of creating the hole.

#### III. Experimental Procedures

#### Specimen Description

Twelve specimens with the geometry shown in Figure 2 were supplied by the Air Force Flight Dynamics Laboratory. They were machined from a single 16-ply, Thornel 300/5208, graphite-epoxy panel fabricated by Monsanto Research Corporation. The multiple layers are arranged symmetrically about the middle surface at orientations of 0, 45, -45, 90, 90, -45, 45, 0 degrees to the x-axis starting with the top ply and progressing to the middle surface. These orientations are repeated for the remaining eight plies below the middle surface. This layup can be expressed as  $((0/\frac{+}{45}/90)_s)_2$ . Rectangular fiberglass doublers, .118 inch thick, with one edge beveled at 30 degrees were adhesively bonded to the specimens. A circular hole was drilled in the center of each specimen. Typical dimensions of the specimens were one inch wide, 0.085 inch thick, eight inches long, with a center hole diameter of 0.20 inch. An aluminum beam of the same geometry as the specimens was used to develop the testing procedures as was an unusable specimen, AB 70. After mounting, the effective length of each specimen from root to tip was 5.12 inches as defined in Figure 2.

#### Constant Amplitude Cycles

Each specimen was taken to failure where failure for this experiment was defined as extensive visual fiber delamination accompanied by a significant change in resonant frequency. Stress levels were chosen so that failure would occur between  $10^4$  and  $10^6$  cycles. From

Figure 2. Specimen Geometry

these stresses, corresponding tip amplitudes were calculated from Eq (21). Constant tip amplitude was maintained in order to keep a constant maximum stress at the root of the beam while at resonance. Resonance was found by observing the peaking of beam response with varying excitation frequency on an oscilloscope.

An M-B electrodynamic vibration exciter, model CIOE, was used for all tests. As the schematic in Figure 3 shows, it was driven by an M-B automatic vibration exciter control, model N695/696, via an M-B power amplifier. The moving element of the exciter was fitted with an aluminum fixture that served as the mount for each specimen. Figure 4 illustrates this arrangement. An Endevco piezoelectric accelerometer, model 2235C, was glued onto the specimen free end and provided feedback to the control unit through two Endevco preamplifiers, models 2614B and 2621. The control unit integrated the acceleration twice and drove the shaker with a sinusoidal signal so as to shake the specimens vertically and maintain the specimens at constant tip amplitude at resonance. This amplitude was read and monitored directly from the control unit. same accelerometer and preamplifiers sent a signal to a Tektronix oscilloscope, model 465M, which monitored the free end acceleration. Another accelerometer of the same type was mounted on the aluminum block at the clamped end of the specimens. Its signal was fed into an M-B vibration meter, model N524, from which base displacement was monitored.

A CMC universal counter-timer, model 726C, monitored the driving frequency continuously. Cycles were counted using this resonant frequency for each beam. This frequency and the time of each

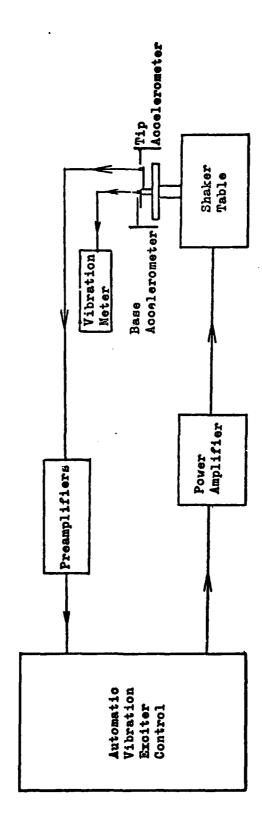


Figure 3. Schematic of Wibration Test System

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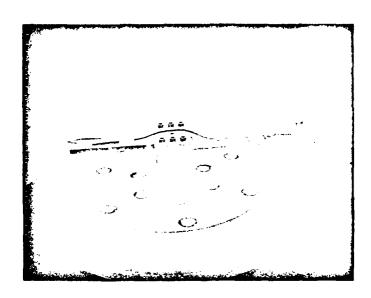


Figure 4. Beam Mounted on Vibration
Exciter Head

test run were used to determine the number of cycles applied in each specimen.

Twisting was not pronounced enough to be noticed in the system.

Logarithmic Decrement

In a system with  $\eta<1$ , the degree of damping can be defined in terms of successive peak values of a tip amplitude as a function of time obtained in free vibration. For any two maxima separated by n cycles of oscillation as shown in Figure 5, the ratio of these two maxima is

$$\frac{x_n}{x_0} = e^{-2\pi\eta n/(1-\eta^2)^{\frac{1}{2}}}$$

from which  $\eta$  can be expressed as

A Honeywell visicorder, model 1508, using a Honeywell galvonometer, M-40-350A, recorded the free vibration response of the specimen tip each time a damping measurement was made by logarithmic decrement. This response signal was provided to the visicorder by the tip accelerometer through the preamplifiers. The damping measurements using logarithmic decrement were all taken at a root stress level of  $\sigma_R$  = 16450 psi. Bandwidth

In the bandwidth method, damping is indicated by the width of the

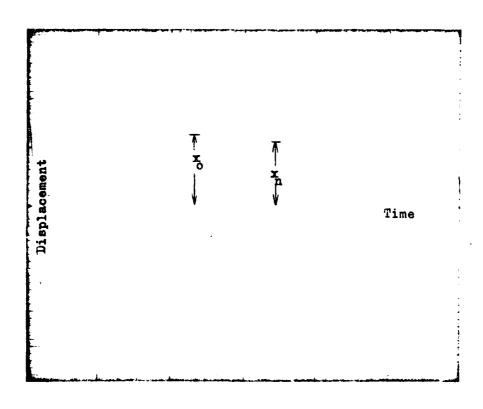


Figure 5. Trace of Damped Free Vibration

response curve near the resonant frequency,  $\nu_{\rm R}$ , as shown in Figure 6. Designating the width as a frequency increment,  $\Delta\nu$ , measured at the half-power points (at a value of  ${\rm R=R}_{\rm max}/\sqrt{2}$ ), the damping of the system is defined as  $=\frac{\Delta\nu}{2\nu_{\rm R}}$  for values of  $\eta<0.1$ .

The tip accelerometer through the preamplifiers provided a signal to the oscilloscope to monitor the free end response of each specimen. This response was recorded in order to determine damping by the bandwidth method as frequency varied with constant base displacement.

Tables III through XIII illustrate the intervals at which damping measurements were made for each specimen using both bandwidth and logarithmic decrement. Initially, logarithmic decrement only was used to calculate damping. The bandwidth technique was not incorporated until the reading at 10<sup>5</sup> cycles since the specimens had to be vibrated during the bandwidth measurements. At 10<sup>5</sup> cycles and subsequent intervals of measurement, the number of cycles put on the beam during the measurement were negligible compared to the total number of cycles on the beam. Bandwidth damping measurements were taken at a representative root stress level which occurred at the stress level midway between the root stress at the peak response and the root stress at the half-power points of the response curve.

#### NDT

The Air Force Materials Laboratory performed nondestructive tests on the specimens after the fatigue tests to determine the extent of damage. Radiographic inspection was performed using a Norelco 150 KVP unit. The specimens were treated with tetrabrome-

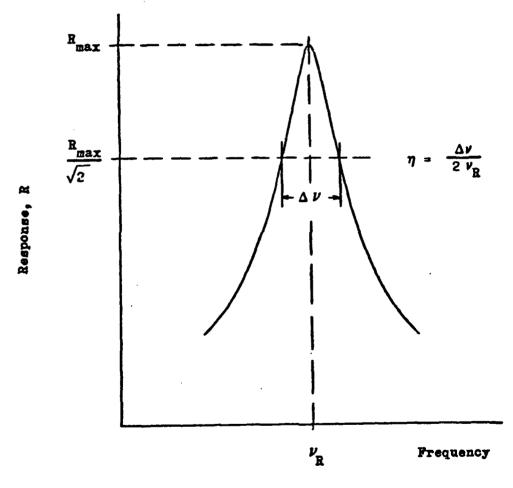


Figure 6. Response Curve Showing Bandwidth at Half-Power Points

thane on both front and back sides for thirty minutes prior to conducting the x-ray inspection. X-rays were taken at  $0, \pm 15$  degrees from the perpendicular.

Ultrasonic C-scans were taken by a Sperry UM 721 Reflectoscope.

The ultrasonic frequency range of the instrument amplifier was

ten to twenty megahertz. The ultrasonic attenuation of the specimens
was four decibels with respect to water.

#### IV. Results and Discussion

#### Bending Fatigue

Results of the bending fatigue tests are presented in Table II. The stress, Tmax, is the stress computed to exist at the edge of the hole from the observed tip displacement. The fatigue data in terms of maximum root stress have been plotted in the form of an S-N curve in Figure 7. The line drawn through the experimental points represents the best fit by visual means. Tests on specimens AB 96 and AB 98 were terminated due to mechanical failure. AB 70 was not tested due to the extensive damage present prior to the experiment.

Single, continuous logitudinal cracks were observed to initiate at the hole and progress to the root of each specimen. The width of the damage zone was equal to the diameter of the hole. Figures 8 and 9 illustrate damage in three of the beams which was typical of the damage in each of the specimens. Although not presented here, the x-rays taken at ±15 degrees from the perpendicular indicated that the damage was not limited to the outer layer.

#### Damping

Material damping data were taken at selected cycle increments and calculated from logarithmic decrement and bandwidth. As the number of cycles on each beam increased, the damping increased as expected due to the permanent microstructural damage occurring in the specimens. The damping results show the same general pattern of increase for each specimen regardless of root stress level and total number of cycles applied. Figure 10 typically illustrates how the

Table II

Specimen	Maximum Stress at Hole, $\sigma_{ ext{max}},$ psi	Root Stress, $\sigma_R$ , psi	Cycles to Failure
AB 74	47850	50700	1.07 x 10 <sup>6</sup> *
AB 77	51530	54600	$6.83 \times 10^5$
AB 81	63500	67410	$9.63 \times 10^4$
AB 82	70120	74660	$3.96 \times 10^4$
AB 90	74110	78560	$3.01 \times 10^5$
AB 93	64430	68530	$2.39 \times 10^5$
AB 94	58150	61840	$6.93 \times 10^5$
AB 95	75960	80780	$1.42 \times 10^5$
AB 96**	53430	56830	
∆B 98**	53960	57380	
\В 99	51860	55160	$2.80 \times 10^6$

<sup>\*</sup> Failure not observed; test terminated

<sup>\*\*</sup> Test terminated due to mechanical failure

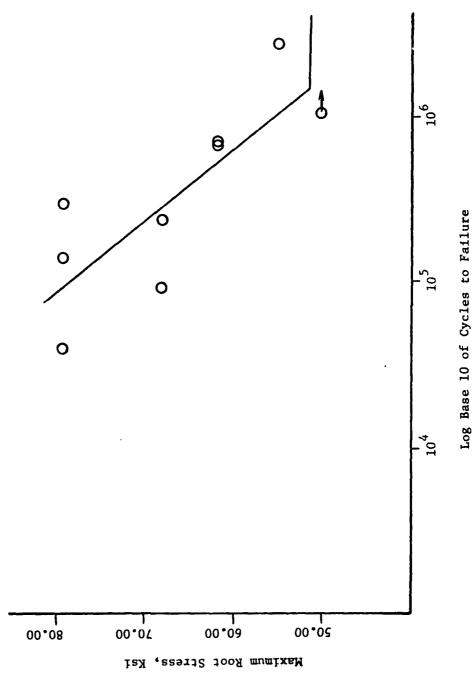
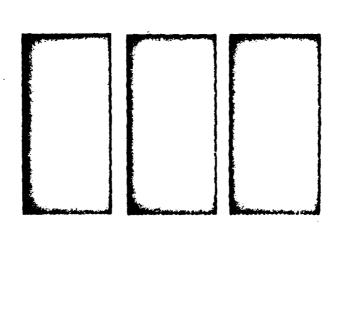


Figure 7. S-N Plot



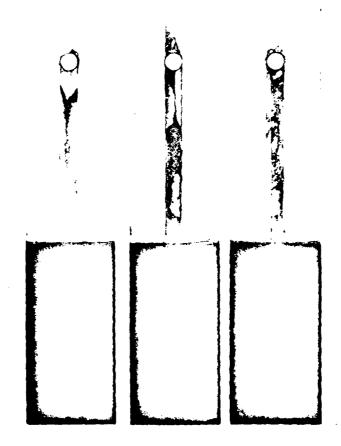


Figure 8. X-rays of Typical Failure

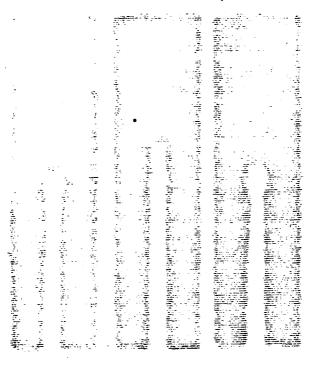
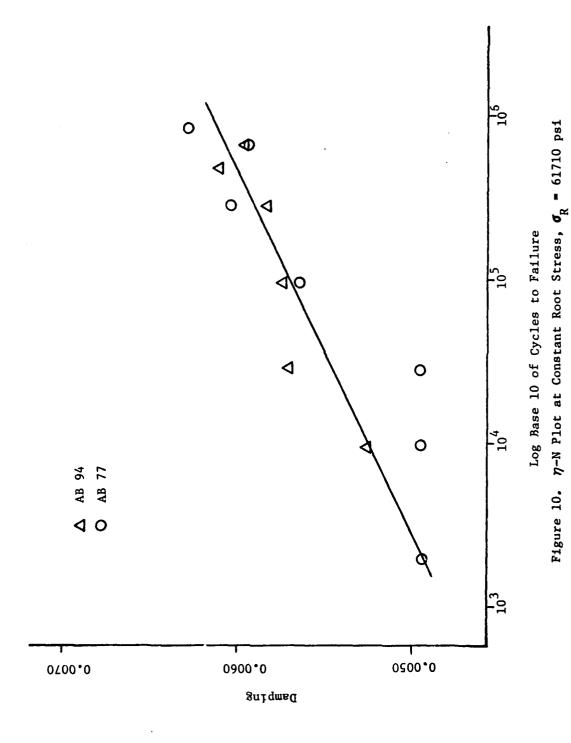


Figure 9. Ultrasonic C-scans of Typical Failure



damping changed as the number of cycles increased. Each damping value plotted was calculated by logarithmic decrement at the same stress level,  $\sigma_{\rm R}$ =16450 psi. Logarithmic decrement and bandwidth damping values for all the beams are tabulated in Tables III through XIII. The values in parentheses next to bandwidth in these tables are root stresses in psi at which the bandwidth measurements were made.

An experiment was performed in order to determine how much of the total damping was material damping and how much was air damping. As shown in Figure 11, an aluminum paddle was attached to the free end of one of the specimens, centered on the beam and flush with the tip; the total surface area was increased by a factor of two. Free vibration measurements were taken of the beam by itself and then of the beam with the paddle. Air damping was determined as explained in Appendix C. Results show that air damping was present but was only on the order of five percent of the total damping. Therefore, the damping results are primarily indicative of material damping behavior.

In general, the bandwidth and logarithmic decrement damping values were in fair agreement. This may be due to taking damping measurements at different stress levels; bandwidth measurements were higher and taken at higher stress levels than those measured by logarithmic decrement.

For each logarithmic decrement curve obtained, damping values varied as the stress decreased while the beam damped out in free vibration.

Damping calculated by logarithmic decrement increased as a greater root stress was applied at the same number of cycles.

Table III

Specimen AB74	$\sigma_{\rm R}$ = 50700 psi	Rel Hum = 55% $T = 74^{\circ}F$	
N	Log Dec	Bandwidth(38730)	
0	.0035		_
10 <sup>5</sup>		.00547	
10 <sup>6</sup>	.00493	.00996	

Table IV

Specimen AB77	$\sigma_{R}$ = 54600 psi	Rel Hum = $52\%$ T = $74^{\circ}$ F
N	Log Dec	Bandwidth(50170)
0	•00424	
104	.00494	
3 x 10 <sup>4</sup>	.00494	
10 <sup>5</sup>	.00563	.00677
3 x 10 <sup>5</sup>	.00602	.00714
6.83 x 10 <sup>5</sup>	.00593	.00604
8.73 × 10 <sup>5</sup>	.00627	

Table V

Specimen AB81	$\sigma_{\rm R}$ = 67410 psi	Rel Hum = 52%	T = 74°F
N	Log Dec	Bandwidth(56	330)
0	.00420		
104	.00426		
3 x 10 <sup>4</sup>	.00565		
9.63 x 10 <sup>4</sup>	.00541	.00660	

Table VI

Specimen AB82	σ <sub>R</sub> = 74660 psi	Rel Hum = 54% T = 67°F
N	Log Dec	Bandwidth(64250)
0	.00270	
104	.00561	
3.96 x 10 <sup>4</sup>	.00605	.00471

Table VII

Specimen AB90	σ <sub>R</sub> = 78560 psi	Rel Hum = 50%	$T = 63^{\circ}F$
N	Log Dec	Bandwidth(63	370)
0	•00379		<del></del>
104	.00542		
3 x 10 <sup>4</sup>	•00553		
2.52 x 10 <sup>5</sup>	.00735		
3.01 x 10 <sup>5</sup>	.00710	.00823	

# Table VIII

Specimen AB93	$\sigma_{\rm R}$ = 68530 psi	Rel Hum = 45%	$T = 82^{\circ}F$
N	Log Dec	Bandwidth(572	210)
0	.00350		
10 <sup>4</sup>	.00509		
3 x 10 <sup>4</sup>	.00515		
10 <sup>5</sup>	.00511		
2.39 x 10 <sup>5</sup>	.00602	.00567	

Table IX

Specimen AB94	$\sigma_{\rm R}$ = 61840 psi	Re1 Hum = $47\%$ T = $70^{\circ}$ F
N	Log Dec	Bandwidth(49290)
0	•00429	
10 <sup>4</sup>	.00525	
3 x 10 <sup>4</sup>	.00570	
10 <sup>5</sup>	.00572	
3 x 10 <sup>5</sup>	.00582	•00529
5.01 x 10 <sup>5</sup>	•00619	•00730
6.93 x 10 <sup>5</sup>	•00595	•00846

Table X

Specimen AB95	σ <sub>R</sub> = 80780 psi	Rel Hum = $43\%$ T = $76^{\circ}$ F
N	Log Dec	Bandwidth (66890)
0	•00443	
104	.00651	
3 x 10 <sup>4</sup>	.00689	
1.42 x 10 <sup>5</sup>	.00710	.00680

Table XI

Specimen AB96	$\sigma_{\rm R}$ = 6830 psi	Rel Hum = 44%	$T = 75^{\circ}F$
N	Log Dec		
0	.00441		
104	.00591		
3 x 10 <sup>4</sup>	.00587		

Table XII

Specimen AB98	<b>σ</b> <sub>R</sub> = 57380 psi	Rel Hum = 41%	$T = 74^{\circ}F$
N	Log Dec	·	
0	.00443		
104	•00516		
3 x 10 <sup>4</sup>	.00523		
10 <sup>5</sup>	.00526		

Table XIII

Specimen AB99	$\sigma_{\rm R}$ = 51860 psi	Re1 Hum = 40%	$T = 81^{\circ}F$
N	Log Dec	Bandwidth(42	250)
0	.00468		
704	.00510		
3 x 10 <sup>4</sup>	.00559		
10 <sup>5</sup>	.00590	.00614	
3 x 10 <sup>5</sup>	.00580	.00688	
10 <sup>6</sup>	.00596	。00522	
2 x 10 <sup>6</sup>	.00572	,00522	
2.80 x 10 <sup>6</sup>	.00580	.00522	

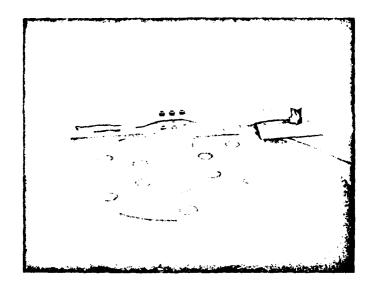
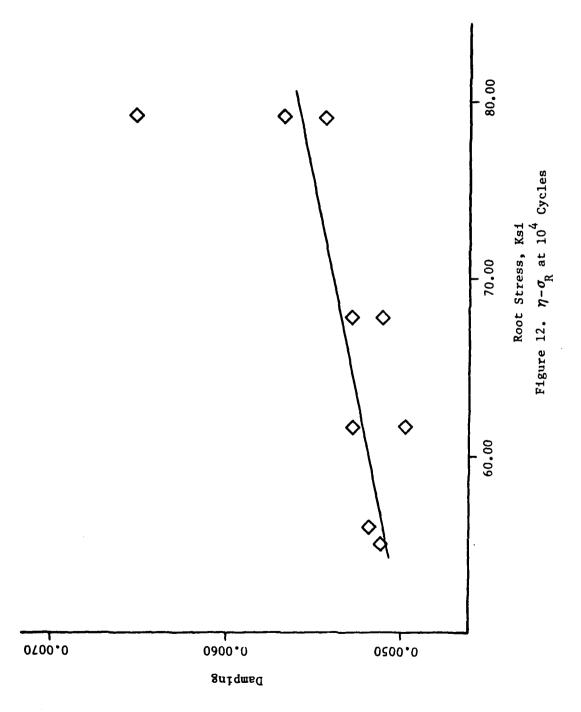


Figure 11. Air Damping Experiment

Figure 12 exhibits typical results of the damping increase with increase in root stress. The same results occurred for each cycle increment at which damping was measured.

## Frequency

For each specimen, the initial resonant frequency dropped to a lower resonant frequency within 3 x 10<sup>3</sup> cycles. Resonance occurred at succeedingly lower frequencies throughout the test runs. Typically, the final resonant frequency was no more than three cycles less than the initial value as shown in Figure 13 and Table XIV. This decrease is due, as is the damping increase, to the permanent damage experienced by the specimens. Since frequency is a measure of stiffness, this reduction indicates a permanent change in the stiffness of the material.



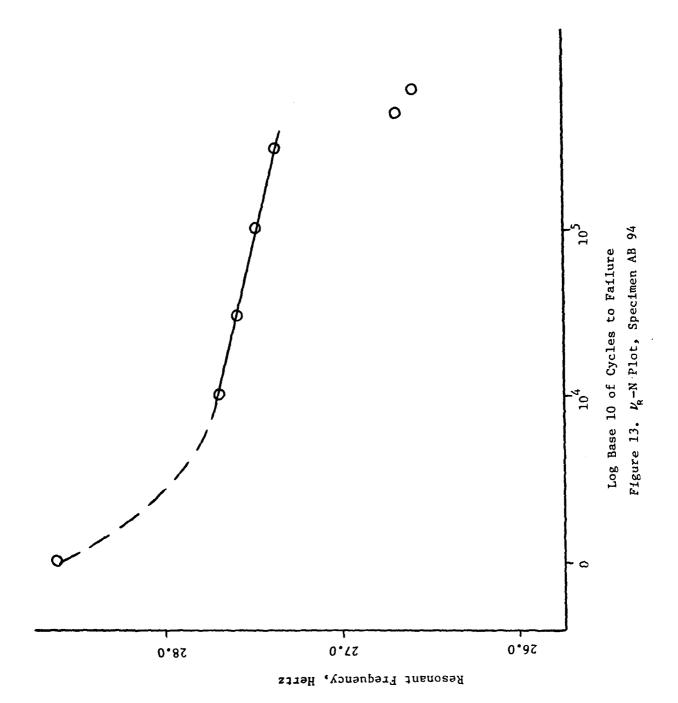


Table XIV

Natural Frequency, CPS						
Specimen	0 Cycles	10 <sup>4</sup> Cycles	10 <sup>5</sup> Cycles	10 <sup>6</sup> Cycles	At Failure	
AB 74	26.1		25.6	25.6	*	
AB 77	27.5	26.8	26.6		25.9	
AB 81	28.2	27.3			26.6	
AB 82	27.9	26.7			25.6	
AB 90	26.6	25.9	25.6		24.9	
AB 93	27.0	26.2	25.8		24.7	
AB 94	28.6	27.7	27.6		26.6	
AB 95	27.8	26.8	25.6		25.0	
AB 96	28.3	27.4			**	
AB 98	26.9	26.1	26.0		**	
AB 99	28.5	27.9	27.7	26.8	26.8	

<sup>\*</sup> Test terminated; no failure observed

<sup>\*\*</sup> Test terminated due to mechanical failure

#### V. Conclusions

# It is concluded that:

- 1) Test results indicate that it is feasible to quantify damage in composites in terms of changes in material damping; material damping increased as damage occurred in the graphite-epoxy specimens.
- 2) Frequency decreases as damage increases. This a measure of the material stiffness which may also prove to be a valid measure of damage.
- 3) Measured material damping is dependent on stress level; nigher stresses yield large damping values.
- 4) Air damping, though present, was not a significant portion of the measured damping. Therefore, the damping values reflect a true change in the material damping.

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#### Appendix A

## Development of the Composite Stiffness

In order to determine the stiffness of the composite, it is necessary to use the moment equation:

$$M_{x} = \int_{-c}^{c} \sigma_{x} z dx$$

where M is the moment per unit length as shown in Figure A-1 and  $\sigma_{\rm x}$  can be expressed in terms of the laminate middle surface strains,  $\epsilon_{\rm x}^{\rm O}$ , and curvatures,  $\kappa_{\rm x}$ 

$$\sigma_{x} = \left[ \overline{Q}_{11} \right]_{k} \left\{ \epsilon_{x}^{\circ} \right\} + z \left\{ \kappa_{x} \right\}$$

Then the moment can be written in terms of this stress-strain relationship:

$$M_{\mathbf{x}} = \sum_{k=1}^{N} \left[ \overline{Q}_{11} \right] k \left( \int_{\mathbf{Z}_{k-1}}^{\mathbf{Z}_{k}} z \, dz + \int_{\mathbf{Z}_{k-1}}^{\mathbf{Z}_{k}} \left\{ \kappa_{\mathbf{x}} \right\} z^{2} \, dz \right)$$

where  $Q_{ij}$  are transformed reduced stiffnesses and  $z_k$ ,  $z_{k-1}$  are defined in Figure A-2. Since  $\epsilon_x^0$  and  $\kappa_x$  are not functions of z but are middle surface values, they can be removed from under the summation sign. Then this equation can be written as

$$\mathbf{M}_{\mathbf{x}} = \begin{bmatrix} \mathbf{B}_{11} \end{bmatrix} \left\{ \boldsymbol{\epsilon}_{\mathbf{x}}^{\circ} \right\} + \begin{bmatrix} \mathbf{D}_{11} \end{bmatrix} \left\{ \boldsymbol{\kappa}_{\mathbf{x}} \right\}$$
where  $\mathbf{B}_{\mathbf{i}\mathbf{j}} = \frac{1}{2} \sum_{k=1}^{N} (\overline{\mathbf{Q}}_{\mathbf{i}\mathbf{j}})_{k} \quad (\mathbf{z}_{k}^{2} - \mathbf{z}_{k-1}^{2}) \quad \text{and}$ 

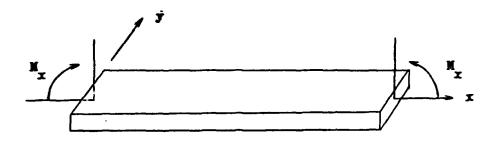


Figure A-1. Moment on Laminate Beam

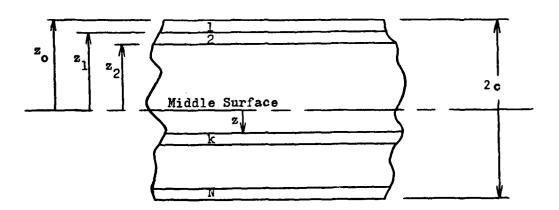


Figure A-2. Geometry of an N-layered Laminate

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N} (\overline{Q}_{ij})_{k} (z_{k}^{3} - z_{k-1}^{3})$$

 ${f D}_{ij}$  are the bending stiffnesses and  ${f B}_{ij}$  are the coupling stiffnesses and imply coupling between laminate bending and extension. Therefore since the beams are assumed to undergo only bending, all  ${f B}_{ij}$  = 0. Then the moment equation becomes

$$M_{x} = \left[D_{11}\right]\left\{\kappa_{x}\right\}$$

where 
$$D_{11} = \frac{1}{3} \sum_{k=1}^{N} (\overline{Q}_{11})_k (z_k^3 - z_{k-1}^3)$$

The engineering constants for the beams are

$$E_{11} = 2.3 \times 10^7 \text{ psi}$$

$$E_2 = 1.6 \times 10^6 \text{ psi}$$

$$G_{12} = (.7-.8) \times 10^6 \text{ psi}$$

$$\nu_{12} = .3$$

$$v_{21} = .01$$

Then the reduced stiffnesses become

$$Q_{11} = \frac{E_1}{1 - \nu_{12} \nu_{21}} = 2.3069 \times 10^7 \text{ psi}$$

$$Q_{12} = \frac{\nu_{12} E_2}{1 - \nu_{12} \nu_{21}} = 4.8144 \times 10^5 \text{ psi}$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12} \quad \nu_{21}} = 1.6048 \times 10^6 \text{ psi}$$

$$Q_{66} = G_{12} = .75 \times 10^6 \text{ psi}$$

Only one transformed reduced stiffness,  $\overline{\mathbb{Q}}_{11}$ , is needed to calculate the stiffness. It is related to the reduced stiffness in the following manner:

$$\overline{Q}_{11} = Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66})\sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta$$

where  $\theta$  = angle from the x-axis to the axis the fibers are directed along.

For the 0 degree plies,  $\overline{Q}_{11} = 2.3069 \times 10^7 \text{ psi}$ .

For the  $\frac{+}{4}$  45 degree plies,  $\overline{Q}_{11} = 7.1592 \times 10^6$  psi.

For the 90 degree plies,  $\overline{Q}_{11} = 1.6048 \times 10^6 \text{ psi}$ .

Then  $D_{11} = 5.8233 \times 10^2 \text{ lb-in}^2$ .

Since  $M_x = EI \frac{\partial^2 w}{\partial x^2} = EI K_x$  then

EI 
$$\kappa_x = D_{11} \kappa_x$$
 or EI =  $D_{11}$ 

and EI =  $5.8233 \times 10^2 \text{ lb-in}^2$ .

For a beam, the moment of inertia is  $I = \frac{bh^3}{12}$ .

Using the typical dimensions b = 1 inch and h = .085 inch for

these specimens,  $I = 5.1177 \times 10^{-5} \text{ in}^4$ .

The effective Young's Modulus becomes  $E = 1.138 \times 10^7$  psi for this composite.

# Appendix B

## Material Stiffness

In order to verify the calculation of stiffness for the composite specimens, an attempt was made to measure the stiffness experimentally.

Weights were suspended from the free end of a cantilevered specimen and the tip deflection was recorded. The stiffness was then calculated from

$$\delta_{\rm T} = \frac{\rm P1}{\rm 3EI}^{\rm 3}$$

where P = force applied at distance, 1, from the root. Results are
presented in Table XV.

Table XV
Static Stiffness

•06	378.83
.12	378.83
.18	378.83
.24	378.83
	.12

The measured values are in poor agreement with the computed value of EI = 582.33 lb-in. In view of the fact that the computed stiffness led

to a predicted resonant frequency of  $\nu_{\rm R}$  = 27.85 cps which was in much better agreement with measured values (26.1 - 28.6 cps) than the value of  $\nu_{\rm R}$  = 20.42 cps calculated using Eq (9a) and the measured stiffness, the computed EI was accepted as being correct.

#### Appendix C

In order to determine how much of the total damping was air damping, an aluminum paddle, three inches square, was attached to a specimen, centered and flush with its tip. This increased the surface area by a factor of two. Free vibration measurements were made of the beam with the paddle and of the beam alone. The resonant frequency was approximately the same for both configurations.

Air damping is proportional to the area of the vibrating object and the square of the maximum amplitude. Linearizing the mode shapes of both configurations led to approximating the air damping in terms of an average amplitude. For the beam alone, expressed in terms of the tip amplitude, this was  $0.5\delta_{\rm T}$ . For the beam and paddle, the average amplitude was  $0.8\delta_{\rm T}$ . This yielded a damping value for the beam and paddle system six times that of the beam alone.

The measured damping was greater for the beam and paddle than for the beam. Assuming material damping was the same for both cases, it was eliminated from the difference of the two measured values and the air damping was solved for. The air damping was on the order of five percent of the total damping of the beam. Therefore, the damping results in this study are indicative of material damping behavior.

#### <u>Vita</u>

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION I	PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFIT/GA/AA/78D-2	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle)		5. TYPE OF REPORT & PERIOD COVERED
MATERIAL DAMPING AS A MEANS OF	QUANTIFYING	MS Thesis
FATIGUE DAMAGE IN COMPOSITES		6. PERFORMING GRG. REPORT NUMBER
7. AUTHOR(s)		8. CONTRACT OR GRANT NUMBER(a)
Cynthia A. Bourne Second Lieutenant		
9. PERFORMING ORGANIZATION NAME AND ADDRESS		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
Air Force Institute of Technolo Wright-Patterson AFB, Ohio 45		
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE
Air Force Flight Dynamics Labor	atory	December 1978
Wright-Patterson AFB, Ohio 45	433	13. NUMBER OF PAGES 57
14. MONITORING AGENCY NAME & ADDRESS(II different	from Controlling Office)	15. SECURITY CLASS. (of this report)
		Unclass for
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)		<u> </u>
*7. DISTRIBUTION STATEMENT (of the abstract entered in	n Block 20, if different from	n Report)
	oved for public	release; IAW AFR 190-17
Dire	ctor of Informat	ion
19. KEY WORDS (Continue on reverse side II necessary and Composite Materials Graphite-Epoxy	identify by block number)	
Bending Fatigue Test		
Damping		+ 600
Damage Assessment  20. ABSTRACT (Continue on reverse side if necessary and	identify by block number)	
The purpose of this study wa bility of quantifying damage in characteristics. Bending fatige T300/5208 graphite-epoxy composes	s to experiment composites in que tests were p	terms of their damping performed on ((0/45/90))
by exciting a cantilever beam so	as to maintain	

composite can be explained in terms of an increase in its material damping. The resonant frequency (stiffness) was found to decrease as the material damage increased.					
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